

CALCULUS II ASSIGNMENT 7

DUE MARCH 14, 2019

An infinite number of calculus students walk into a bar.

The first says, "I will have a beer."

The second says, "I will have half a beer."

The third says, "I will have a quarter of a beer."

Before anyone else can speak, the bartender fills up exactly two glasses with beer and serves them.

"Come on, now," says the bartender, "you have got to learn your limits."

Okay, now time to heed the good bartender's advice...

1. Use the Integral Test to determine whether the series is convergent or divergent:

(i) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$,

(iii) $\sum_{k=1}^{\infty} k e^{-k}$,

(ii) $\sum_{m=1}^{\infty} \frac{1}{n\sqrt{2}}$,

(iv) $\sum_{\ell=1}^{\infty} \frac{2}{5n-1}$.

2. The harmonic series $\sum \frac{1}{n}$ is divergent, but only just so. This is perhaps substantiated by the following exercise: determine the values of p for which the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log(n)^p}$$

converges.

Incidentally, I just learned **this amazing fact** that if you throw out the terms of the series with "9" in the number, then the resulting series will converge! I think that's pretty awesome.

3. What does the decimal representation of a real number really mean? Say we write a number as $0.d_1 d_2 d_3 d_4 \dots$, where d_i is one of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This is really a short hand for the series

$$0.d_1 d_2 d_3 d_4 \dots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \frac{d_4}{10^4} + \dots$$

To make sense of this statement, show that for any choice of $d_1, d_2, d_3, d_4, \dots$, the series on the right is convergent.

Try to generalize this to where the denominators on the right are powers of a number different than 10 and where the digits d_i perhaps range through a different set of values.

This allows you to make sense of expansions of numbers in different **base systems**. Not *that* **base system**, though...