# CALCULUS II ASSIGNMENT 6

DUE MARCH 12, 2019

This xkcd comic would have served well for the introduction of this course. In any case, time to continue on the series...

**1.** Justify why the following series converge and find their sums.[1],

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4+n^2}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$,

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$, (iv) $\displaystyle\sum_{n=1}^{\infty} \big(\sin(1/n)-\sin(1/(n+1))\big)$.

**2.** Determine whether or not the following series converge or diverge. If they converge, determine their sum.

(i) $\displaystyle\sum_{n=1}^{\infty} \cos(n)$, (iv) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2-2n+5}$,

(ii) $\displaystyle\sum_{k=1}^{\infty} \sin(100)^k$, (v) $\displaystyle\sum_{i=1}^{\infty} \frac{3^{i+1}}{(-2)^i}$,

(iii) $\displaystyle\sum_{m=2}^{\infty} \frac{1}{m^3-m}$, (vi) $\displaystyle\sum_{\ell=1}^{\infty} \frac{1}{1+(2/3)^{\ell}}$.

**3.** The Comparison Test we discussed in class says that given two series $\sum a_i$ and $\sum b_i$ with positive terms, then

(i) if $\sum b_i$ is convergent and $a_n \le b_n$ for all sufficiently large indices $n$, then $\sum a_i$ is convergent; and

(ii) if $\sum b_i$ is divergent and $a_n \ge b_n$ for all sufficiently large indices $n$, then $\sum a_i$ is divergent.

Formulate analogues of statements (i) and (ii) for series $\sum a_i$ and $\sum b_i$ with negatives terms. Try to justify your statements using (i) and (ii) above.

**4.** Use the Comparison Test to determine whether or not the following series converge or diverge:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$, (iv) $\displaystyle\sum_{\ell=1}^{\infty} \frac{e^{1/\ell}}{\ell}$,

(ii) $\displaystyle\sum_{m=1}^{\infty} \frac{\log(m)}{m}$, (v) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$,

(iii) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^k}$, (vi) $\displaystyle\sum_{m=1}^{\infty} \frac{9^m}{3+10^m}$.

[1]March 7: I misjudged **1.**(i) and the sum is not so easy to compute. Simply justify why it converges, please!

In some of the comparisons above, it might be useful to know that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges when $p > 1$ and diverges when $p \leq 1$; note that the $p = 1$ case is the Harmonic Series, which I mentioned in class. We will see why these statements are true, soon!