CALCULUS II ASSIGNMENT 4

DUE FEBRUARY 21, 2019

Let's do some practice for the midterm. Most of this amounts to practice with the techniques we have developed over the last month.

1. Evaluate the integrals

(i) $\displaystyle\int \frac{e^{\arctan(y)}}{1+y^2}\,dy$,

(ii) $\displaystyle\int_0^\pi t\cos(t)\,dt$,

(iii) $\displaystyle\int_0^1 \left(1+\sqrt{x}\right)^8\,dx$,

(iv) $\displaystyle\int \sqrt{u}e^{\sqrt{u}}\,du$,

(v) $\displaystyle\int \frac{\sec(\theta)^2}{\tan(\theta)^2-16}\,d\theta$,

(vi) $\displaystyle\int \phi\tan(\phi)^2\,d\phi$,

(vii) $\displaystyle\int x\arctan(x)\,dx$,

(viii) $\displaystyle\int \frac{\sin(\psi)\cos(\psi)}{\sin(\psi)^4+\cos(\psi)^4}\,d\psi$.

2. Compute

$$\int_0^{\pi/2} \sin(x)^{2n}\,dx$$

for $n = 0,1,2,3$. Can you guess what the value of the integral might be for arbitrary n?

3. Evaluate the integral

$$\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx$$

by completing the square in the denominator and doing a trigonometric substitution.

Okay, I think if you can do all the integrals above, then you are in more than good shape for the midterm. The remainder of this set is culture in something that I think is pretty cool; I hope you have some fun with it!

4. Let's think about the calculations that we performed in Assignment 2, Question **2**. A function of the form

$$f(x) = a_1\sin(x) + a_2\sin(2x) + \cdots + a_N\sin(Nx)$$

is called a *finite Fourier series*. Show that

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(mx)\,dx.$$

Moreover, verify that

$$\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx = 0$$

for all positive integers m. This gives a way of extracting the coefficients a_i in a function of this form!

5. The calculation in **3.** suggests a way to approximate certain functions. Consider the function

$$f(x) := \begin{cases} -1 & \text{if } -\pi \le x < 0, \\ 1 & \text{if } 0 \le x \le \pi. \end{cases}$$

(i) Calculate

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(mx)\, dx$$

for $m = 1, 2, 3, 4, 5, 6, 7, 8$.

(ii) Let

$$f_i(x) = a_1 \sin(x) + a_2 \sin(2x) + \cdots + a_{2i-1} \sin((2i-1)x) + a_{2i} \sin(2ix).$$

With the help of a computer, graph the functions f_1, f_2, f_3, and f_4 in the interval $[-\pi, \pi]$. Also draw the graph of f.