CALCULUS II ASSIGNMENT 3

DUE FEBRUARY 14, 2019

1. Let's do a few more trigonometric substitutions, just to get a feel for what sort of integration problems they might be useful for:

(i) Use the substitution $y = 3\sin(\phi)$ to compute $\int \sqrt{9-y^2}\,dy$.

(ii) Use the substitution $x = 2\tan(\theta)$ to compute $\int \frac{1}{x^2+4}\,dx$.

(iii) Use the substitution $z = 5\sec(\psi)$ to compute $\int \frac{1}{\sqrt{z^2-25}}\,dz$.

2. Here is some practice for integrals of rational functions:

(i) $\int \frac{x^4}{x-1}\,dx$,

(ii) $\int \frac{y}{(y-3)(2y+1)}\,dy$,

(iii) $\int \frac{t^2+t+2}{t^2-1}\,dt$,

(iv) $\int \frac{1}{z^2-2z}\,dz$,

(v) $\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$,

(vi) $\int \frac{u+2}{u^4+3u^3+3u^2+u}\,du$,

(vii) $\int \frac{1}{1+e^y}\,dy$,

(viii) $\int \frac{4v+2}{v(v^2+1)^2}\,dv$.

You may have needed to factor a quadratic polynomial, perhaps using the quadratic formula...

3. Let's do something fun with polar coordinates!

(i) Sketch the curve defined in polar coordinates by

$$r = 1 - \cos(\theta).$$

Feel free to ask your computer for help.

(ii) Compute the integral

$$A = \frac{1}{2}\int_0^{\pi} r^2\,d\theta$$

where r is the function of θ defined in (i).

(iii) Informally explain in your own words why the quantity $2A$ is the area of the figure drawn in (i). It may be helpful to know the area of the sector of a circle is $\frac{1}{2}r^2\theta$. For somewhat a formal explanation, see here.

This figure is called a cardiod. I think it's rather pretty. Happy Valentines Day!