# CALCULUS II ASSIGNMENT 11

DUE APRIL 25, 2019

**1.** Perhaps a somewhat enlightening way to solve second-order linear differential equations is to use some sort of series method. We will solve the equations

$$y'' = y \quad \text{and} \quad y'' = -y$$

using this method.

(i) Assume that the equation $y'' = y$ admits a solution $f$ which is a function with a Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Use the equation $f'' = f$ to get a relationship between the coefficients $c_n$ and $c_{n-2}$.

(ii) Use the relationships from (i) to express $c_{2n}$ in terms of $c_0$ and $c_{2n+1}$ in terms of $c_1$.

(iii) Use (ii) to recognize that the power series for $f$ must be obtained as a linear combination of the power series of $e^x$ and $e^{-x}$, yielding the general solution to $y'' = y$.

(iv) Apply the same method to solve $y'' = -y$.

**2.** Find the following areas. It may be helpful to sketch out what the regions look like.

(i) $f(x) = \sin(x)$ and $g(x) = x$ on the interval $[\pi/2, \pi]$.

(ii) The area enclosed by $f(x) = x^2$ and $g(x) = 4x - x^2$.

(iii) The region enclosed by $f(x) = \frac{x}{1+x^2}$ and $g(x) = \frac{x^2}{1+x^3}$.

(iv) The area enclosed by $f(x) = 1/x$, $g(x) = x$, $h(x) = x/4$ when $x > 0$.

**3.** Find the volume of the following solids of rotation:

(i) Spin the region $y = x + 1$, $y = 0$, $x = 0$, and $x = 2$ about the $x$-axis.

(ii) Rotate $y = e^x$, $y = 0$, $x = -1$, and $x = 1$ about the $x$-axis.

(iii) Turn $y = x^2$ and $x = y^2$ around the line $y = 1$.

(iv) Gyrate $y = x^3$, $y = 0$, and $x = 1$ about the line $x = 2$.