CALCULUS II ASSIGNMENT 1

DUE JANUARY 31, 2019

1. Compute the following integrals using substitution:

(i) $\displaystyle\int_0^1 (2+4x)^3\,dx$, (iii) $\displaystyle\int_1^e \frac{\log(z)^4}{z}\,dz$,

(ii) $\displaystyle\int e^t\cos(e^t)\,dt$, (iv) $\displaystyle\int \frac{1+y}{1+y^2}\,dy$.

A function $f(x)$ is called

- *even* if $f(-x) = f(x)$; and
- *odd* if $f(-x) = -f(x)$.

These properties reflect certain symmetries you can find in the graph of f:

(i) if f is even, then its graph is symmetric about the y-axis;
(ii) if f is odd, then the part of the graph left of the y-axis is an upside down version of that to right of the y-axis.

As the next two problems will try to demonstrate, these symmetry properties of functions help you evaluate some definite integrals that might otherwise be difficult.

2. Consider the function $f(x) = x^4$.

(i) Check that $f(x)$ is an even function.
(ii) Sketch a graph of $f(x)$ on the interval $[-2,2]$.
(iii) Compute the integral $\displaystyle\int_{-2}^0 x^4\,dx$ by substituting $u = -x$.
(iv) Use (iii) to compute the integral $\displaystyle\int_{-2}^2 x^4\,dx$.

This is definitely an easy enough example that you can just do directly, but maybe this indicates why observing that your function is even could be labour saving when doing more complicated integrals.

3. Consider the function $f(x) = \dfrac{\tan(x)}{1+x^6+x^{42}}$.

(i) Check that $f(x)$ is an odd function.
(ii) Mimic **2.**(iii) and **2.**(iv) to compute

$$\int_{-3}^3 \frac{\sin(x)}{1+x^6+x^{42}}\,dx = 0.$$

Yes, seeing an odd function is odd is typically much more helpful than seeing an even function.

4. Evaluate the following integrals using integration by parts:

(i) $\displaystyle\int x\sin(4x)\,dx$, (iii) $\displaystyle\int_0^1 \frac{y}{e^{3y}}\,dy$,

(ii) $\displaystyle\int \log(s)^2\,ds$, (iv) $\displaystyle\int e^{2\theta}\sin(3\theta)\,d\theta$.

5. Evaluate the following integrals by first making a substitution and then integrating by parts:

(i) $\displaystyle\int x\log(5+7x)\,dx$, (ii) $\displaystyle\int \sin(\log(x))\,dx$.